FERMILAB-Pub-04/158-E September 2004

Neutral currents and tests of three-neutrino unitarity in long-baseline experiments

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Abstract

We examine a strategy for using neutral current measurements in long-baseline neutrino oscillation experiments to put limits on the existence of more than three light, active neutrinos. We determine the relative contributions of statistics, cross section uncertainties, event misidentification and other systematic errors to the overall uncertainty of these measurements. As specific case studies, we make simulations of beams and detectors that are like the K2K, T2K, and MINOS experiments. We find that the neutral current cross section uncertainty and contamination of the neutral current signal by charge current events allow a sensitivity for determining the presence of sterile neutrinos at the 0.10–0.15 level in probablility.

1 Introduction

In recent years a series of exciting experimental results have shown that neutrinos have finite masses and mixings. For a recent review of the status see Ref. [1]. Solar neutrino and atmospheric neutrino results indicate that all three known neutrino flavors (e, μ, τ) participate in neutrino mixing, and hence neutrino oscillations. Consequently, the standard framework to describe the experimental results and analyse neutrino oscillation data is that of three-flavor mixing in which the three flavor eigenstates are related to three mass eigenstates by a 3×3 mixing matrix[2]. The positive signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations from the LSND experiment[3] challenges the three-flavor mixing paradigm[4]. However, the neutrino oscillation interpretation of the LSND observations is yet to be confirmed. Independent of whether or not the LSND results are confirmed by MiniBooNE[5], the three-flavor mixing framework deserves further experimental scrutiny in the coming years. Much of the focus on future experiments so far has been directed to the determination of the 3-mixing angles and the CP-violating phase with long-baseline oscillation experiments[6] and reactor experiments[7]. Of interest in this paper is the measurement of the neutral current, which could allow tests of the unitarity of the 3×3 mixing matrix and thus indirectly probe the existence of sterile neutrinos.

In a three-flavor neutrino model, the sum of the oscillation probabilities $\sum_{y=e,\mu,\tau} P(\nu_x \rightarrow \nu_y)$ is unity. If there are more than three light neutrinos, we know from measurements of the invisible width of the Z [8] that the additional neutrinos must be sterile. If additional light neutrinos mix with the three known flavors we can expect a non-zero oscillation probability to sterile neutrinos, $P(\nu_x \rightarrow \nu_s) \neq 0$. To test the three-flavor neutrino-mixing paradigm it is important to search for a sterile neutrino component within the neutrino flux from natural and manmade sources. Since sterile neutrinos have no strong or electroweak interactions, they cannot be detected directly. However, neutral current (NC) measurements allow $\sum_{y=e,\mu,\tau} P(\nu_x \rightarrow \nu_y)$ to be determined which, by probability conservation, is equal to $1 - P(\nu_x \rightarrow \nu_s)$. Therefore, in principle a NC measurement alone is sufficient to determine $P(\nu_x \rightarrow \nu_s)$. However, in a realistic detector misidentifications of CC and NC events, together with systematic uncertainties on the relevent neutrino interaction cross sections, complicate the analysis.

In this paper we study the use of NC measurements to determine limits on the sterile neutrino content in long-baseline neutrino oscillation experiments. First we consider the sensitivity to the sterile content that might be obtained in a K2K-like[9], T2K-like[10], and MINOS-like[11] experiment with a "perfect" detector and "perfect" beam if there are no systematic uncertainties. We then consider the impact on the sensitivity of event misidentification and systematic uncertainties. Our study is based on a simple simulation of the long-baseline neutrino beams, neutrino interactions [12], and detector responses. We present our results versus event rates and the size of the cross section uncertainty in order to show the dependence on these quantities.

2 Using NC data to determine sterile content

2.1 Formalism

The present and proposed long-baseline neutrino oscillation experiments exploit conventional neutrino beams that are produced by the decays of charged pions in a long channel. This produces a beam which is initially almost entirely ν_μ. Kaons and muons decaying in the channel introduce a small (typically $\sim 1\%$) ν_e component in the neutrino beam. As the neutrino beam travels towards a distant detector its flavor content will evolve. In our analysis we will consider three active neutrinos (ν_e, ν_μ, ν_τ) and one sterile neutrino (ν_s), with oscillation probabilities $P(\nu_\mu \to \nu_x) \equiv P_{\mu x}$, $x = e, \mu, \tau, s$. We begin by considering an oscillation experiment that has an initially pure ν_μ beam with well known neutrino spectrum and flux, and a detector with perfect identification of the produced events. Then the event rates at the far detector will be

$$N_{\rm NC} = N_\mu^0 (1 - P_{\mu s}) \sigma_{\rm NC}/\sigma_\mu \,, \tag{1}$$

$$N_\mu = N_\mu^0 P_{\mu\mu} \,, \tag{2}$$

$$N_e = N_\mu^0 P_{\mu e} \sigma_e/\sigma_\mu \,, \tag{3}$$

$$N_\tau = N_\mu^0 P_{\mu\tau} \sigma_\tau/\sigma_\mu \,. \tag{4}$$

where N_μ^0 is the predicted number of ν_μ CC interactions in the detector in the absence of oscillations and the σ_x denote the interaction cross sections $(\sigma_e, \sigma_\mu, \sigma_\tau)$ for (ν_e, ν_μ, ν_τ) CC interactions and $\sigma_{\rm NC}$ for ν_x NC interactions.

In an ideal experiment $N_{\rm NC}$ determines $P_{\mu s}$ and $N_{\rm CC}$ determines $P_{\mu\mu}$. In practice the presence of ν_e and ν_τ CC events complicates the analysis if these events are not distinguished from NC events. In that circumstance the NC events provide a measure of $1 - P_{\mu s} + \epsilon_e P_{\mu e} + \epsilon_\tau P_{\mu\tau}$, where the factors ϵ_e and ϵ_τ reflect the contaminations.

Probability conservation ($P_{\mu e} + P_{\mu\mu} + P_{\mu\tau} + P_{\mu s} = 1$) can be used to eliminate $P_{\mu\tau}$ or $P_{\mu e}$, but not both. If the beam energy is below the threshold for τ production or the probability $P_{\mu e}$ is small and can be neglected, then $P_{\mu s}$ can still be determined. However, for a realistic detector with particle misidentifications and/or a ν_e component at the far detector that cannot be neglected, the problem of determining $P_{\mu s}$ can be complex but still solvable, as we shall discuss.

In general, let the probability that an event of type x (NC, ν_e CC, ν_μ CC, or ν_τ CC) be identified in the detector as an event of type y be given by ζ_{xy}, where $x, y = (NC, e, \mu, \tau)$ (note that ζ_{xx} is the efficiency for detecting an event of type x). If N_μ^0 is the predicted number of ν_μ CC interactions in the detector in the absence of oscillations, then after including oscillations, detector efficiencies and mis-identifications, and integrating over the energy dependence, the number of measured events of type y will be:

$$N_y = \frac{N_\mu^0}{\sigma_\mu}\left[(1 - P_{\mu s})\sigma_{NC}\zeta_{NC,y} + \sum_{x=e,\mu,\tau} P_{\mu x}\sigma_x\zeta_{xy}\right] , \tag{5}$$

where the interaction cross sections for ν_e CC, ν_μ CC, ν_τ CC, and NC events are given by σ_e, σ_μ, σ_τ and σ_{NC}, respectively.

In Eq. 5 we do not include explicitly a term that accounts for the ν_e contamination of the beam. In practice, this contamination could be added to $P_{\mu e}$ and subtracted from $P_{\mu\mu}$. The contamination is expected to be of order 1% or less, which is small compared to the mis-identification factors, as we subsequently will show. Therefore we neglect this small correction.

2.2 Ignore ν_e's

We consider first the situation in which the ν_e component in the beam at the far detector is so small that ν_e CC interactions can be neglected. In this case we let $P_{\mu e} \to 0$ (it is known to be small, at the 5% level or less from the CHOOZ experiment[13]). It is convenient to define the following two ratios,

$$R_{NC} \equiv \frac{N_{NC}}{\zeta_{NCNC} N^0_{NC}} = (1 - P_{\mu s}) + f_{\mu,NC} P_{\mu\mu} + f_{\tau,NC} P_{\mu\tau} \,, \tag{6}$$

$$R_\mu \equiv \frac{N_\mu}{\zeta_{\mu\mu} N^0_\mu} = f_{NC,\mu}(1 - P_{\mu s}) + P_{\mu\mu} + f_{\tau,\mu} P_{\mu\tau} \,, \tag{7}$$

where

$$N^0_x \equiv \sigma_x N^0_\mu / \sigma_\mu \,, \tag{8}$$

and

$$f_{x,y} \equiv \zeta_{xy}\sigma_x / \zeta_{yy}\sigma_y \tag{9}$$

is a normalized misidentification factor that gives the ratio of the number of events of type x identified as type y to the number of events of type y that are identified as type y. Measuring R_{NC} and R_μ is sufficient for deducing $P_{\mu s}$ (and $P_{\mu\mu}$). The analysis depends on whether or not we are above the ν_τ CC interaction threshold, i.e., whether or not there are ν_τ CC events produced in the detector.

2.2.1 Below τ threshold

For neutrino energies below the τ threshold $\sigma_\tau = 0$ and $f_{\tau,j} = 0$. In this case we can invert Eqs. 6 and 7 to obtain

$$P_{\mu\mu} = \frac{R_\mu - R_{NC} f_{NC,\mu}}{1 - f_{\mu,NC} f_{NC,\mu}} \,, \tag{10}$$

$$P_{\mu s} = 1 - \frac{R_{NC} - R_\mu f_{\mu,NC}}{1 - f_{\mu,NC} f_{NC,\mu}} . \tag{11}$$

Adding uncertainties in quadrature we get

$$\delta P_{\mu\mu} = \frac{\sqrt{(\delta R_\mu)^2 + f^2_{NC,\mu}(\delta R_{NC})^2}}{1 - f_{\mu,NC} f_{NC,\mu}} \,, \tag{12}$$

$$\delta P_{\mu s} = \frac{\sqrt{(\delta R_{NC})^2 + f^2_{\mu,NC}(\delta R_\mu)^2}}{1 - f_{\mu,NC} f_{NC,\mu}} \,, \tag{13}$$

where in the limit of Gaussian statistical uncertainties

$$\delta R_j = R_j \sqrt{\frac{1}{N_j} + \epsilon_j^2}\,, \tag{14}$$

and

$$\epsilon_j \equiv \delta N_j^0 / N_j^0\,. \tag{15}$$

The first term in each δR_j is the usual statistical uncertainty, the second comes from the normalization uncertainty (flux *and* cross section).

Note that the normalized mis-identification factors $f_{\mu,NC}$ and $f_{NC,\mu}$ will be sensitive to the neutrino energy spectrum and the detector technology, and therefore must be evaluated for each experimental setup. Most of the mis-identification terms are suppressed by f^2; if $f \leq 0.1$ then $f^2 \leq 0.01$. If the experimental setup is such that we can ignore all terms of order f^2, (see Table 1) we have

$$\frac{P_{\mu s}}{\delta P_{\mu s}} \simeq \frac{1 - R_{NC} + f_{\mu,NC} R_\mu}{\delta R_{NC}}\,, \tag{16}$$

which measures the significance of the deviation of $P_{\mu s}$ from zero.

In Eq. 16 we have not included the effects of uncertainties of the f factors. By considering sub-samples in our Monte Carlo calculations of the $f_{x,y}$, we estimate the uncertainties from the Monte Carlo to be about 2% for the most significant f factors. This has a small effect since the systematic uncertainty of the NC cross section (i.e., ϵ_{NC}) is expected to be at least three times larger; this is especially true when the uncertainties are added in quadrature. If there is a systematic uncertainty for an f that is not small compared to ϵ_{NC}, in practice it can be incorporated into ϵ_{NC}, weighted by the value of f itself.

For a perfect detector that can identify each event correctly, $f_{x,y} = \delta_{xy}$. In this limit $P_{\mu s} = 1 - R_{NC}$ and

$$\frac{P_{\mu s}}{\delta P_{\mu s}} \simeq \frac{P_{\mu s}}{\sqrt{(1 - P_{\mu s})\frac{1}{\zeta_{NCNC} N_{NC}^0} + (1 - P_{\mu s})^2 \epsilon_{NC}^2}}\,. \tag{17}$$

This ratio depends only on $P_{\mu s}$, the experimental statistics, and the systematic uncertainty on the NC measurement. Thus, Eq. 17 defines the maximum sensitivity that is in principle achievable for a given N_{NC}^0 and ϵ_{NC}.

2.2.2 Above τ threshold

If the neutrino energy is above the τ threshold and there is not a clean signature for ν_τ CC events, we can still deduce $P_{\mu\mu}$ and $P_{\mu s}$ by using the identity $P_{\mu\mu} + P_{\mu\tau} + P_{\mu s} = 1$ to eliminate $P_{\mu\tau}$ in Eqs. 6 and 7 (we are still assuming $P_{\mu e} = 0$), which gives

$$
\begin{aligned}
P_{\mu\mu} &= \frac{R_\mu(1 + f_{\tau,NC}) + R_{NC}(f_{NC,\mu} + f_{\tau,\mu})}{1 + f_{\tau,NC} - f_{\tau,\mu} + f_{\tau,NC} f_{NC,\mu} - f_{\mu,NC}(f_{NC,\mu} + f_{\tau,\mu})} \,, &(18)\\
P_{\mu s} &= 1 - \frac{R_{NC}(1 - f_{\tau,\mu}) - R_\mu(f_{\mu,NC} - f_{\tau,NC})}{1 + f_{\tau,NC} - f_{\tau,\mu} + f_{\tau,NC} f_{NC,\mu} - f_{\mu,NC}(f_{NC,\mu} + f_{\tau,\mu})} \,. &(19)
\end{aligned}
$$

If no other process contaminates the ν_μ CC events (i.e., $f_{j,\mu} = 0$ as appears to be the case for a MINOS-like experiment; see Sec. 3), then

$$
\frac{P_{\mu s}}{\delta P_{\mu s}} \simeq \frac{1 + f_{\tau,NC} - R_{NC} + R_\mu(f_{\mu,NC} - f_{\tau,NC})}{\sqrt{(\delta R_{NC})^2 + (f_{\mu,NC} - f_{\tau,NC})^2 (\delta R_\mu)^2}} \,. \qquad (20)
$$

For a perfect detector, $P_{\mu s}/\delta P_{\mu s}$ is again given by Eq. 17.

2.3 Do not ignore ν_e's

If the ν_e CC interaction rate in the far detector is not negligible (which could be the case if $\sin^2 2\theta_{13}$ is near its upper bound and we want to push the uncertainty in the measurement of $P_{\mu s}$ down to the few per cent level), then we need three measurements to be able to solve for all of the probabilities. The potential measurables are

$$
\begin{aligned}
R_{NC} &\equiv \frac{N_{NC}}{\zeta_{NCNC} N^0_{NC}} = (1 - P_{\mu s}) + f_{\mu,NC} P_{\mu\mu} + f_{e,NC} P_{\mu e} + f_{\tau,NC} P_{\mu\tau} \,, &(21)\\
R_\mu &\equiv \frac{N_\mu}{\zeta_{\mu\mu} N^0_\mu} = f_{NC,\mu}(1 - P_{\mu s}) + P_{\mu\mu} + f_{e,\mu} P_{\mu e} + f_{\tau,\mu} P_{\mu\tau} \,, &(22)\\
R_e &\equiv \frac{N_e}{\zeta_{ee} N^0_e} = f_{NC,e}(1 - P_{\mu s}) + f_{\mu,e} P_{\mu\mu} + P_{\mu e} + f_{\tau,e} P_{\mu\tau} \,, &(23)
\end{aligned}
$$

and, if we are above the ν_τ CC interaction threshold,

$$
R_\tau \equiv \frac{N_\tau}{\zeta_{\tau\tau} N^0_\tau} = f_{NC,\tau}(1 - P_{\mu s}) + f_{\mu,\tau} P_{\mu\mu} + f_{e,\tau} P_{\mu e} + P_{\mu\tau} \,. \qquad (24)
$$

2.3.1 Below τ threshold

Below the ν_τ CC threshold energy the three measurements must be R_μ, R_{NC}, and R_e. Then $f_{\tau,j} = 0$, the $P_{\mu\tau}$ terms drop out, and we can invert Eqs. 21-23 to obtain

$$P_{\mu\mu} = \frac{R_\mu(1 - f_{e,NC}f_{NC,e}) - R_{NC}(f_{NC,\mu} - f_{NC,e}f_{e,\mu}) - R_e(f_{e,\mu} - f_{e,NC}f_{NC,\mu})}{1-f}\,, \quad (25)$$

$$P_{\mu e} = \frac{R_e(1 - f_{\mu,NC}f_{NC,\mu}) - R_{NC}(f_{NC,e} - f_{NC,\mu}f_{\mu,e}) - R_\mu(f_{\mu,e} - f_{\mu,NC}f_{NC,e})}{1-f}\,, \quad (26)$$

$$P_{\mu s} = 1 - \frac{R_{NC}(1 - f_{\mu,e}f_{e,\mu}) - R_\mu(f_{\mu,NC} - f_{\mu,e}f_{e,NC}) - R_e(f_{e,NC} - f_{e,\mu}f_{\mu,NC})}{1-f}\,, \quad (27)$$

where $f \equiv f_{\mu,NC}f_{NC,\mu} + f_{e,NC}f_{NC,e} + f_{\mu,e}f_{e,\mu} - f_{\mu,NC}f_{NC,e}f_{e,\mu} - f_{NC,\mu}f_{\mu,e}f_{e,NC}$. The calculation of the δP's is straightforward; each R term has a statistical and systematic uncertainty given by Eq. 14.

Note that for an idealized detector in which no other processes significantly contaminate ν_e CC events (i.e., $f_{j,e} \simeq 0$) and ν_e CC events do not contaminate ν_μ CC events (i.e., $f_{e,\mu} \simeq 0$), then $P_{\mu e} = R_e$. Since $P_{\mu e}$ is small (of order 0.1 or less, as indicated by current oscillation limits), eliminating terms of order f^2 and $fP_{\mu e}$ in this case will recover the situation where we ignored ν_e (i.e., Eq. 16).

2.3.2 Above τ threshold, no τ measurement

For energies above the ν_τ CC interaction threshold the $P_{\mu\tau}$ terms do not drop out of Eqs. 21–23. If we do not have the means to measure ν_τ CC events but can measure ν_e CC events, then we can use probability conservation to eliminate $P_{\mu\tau}$, giving

$$\begin{aligned} R_{NC} &= (1 - P_{\mu s})(1 + f_{\tau,NC}) + P_{\mu\mu}(f_{\mu,NC} - f_{\tau,NC}) + P_{\mu e}(f_{e,NC} - f_{\tau,NC})\,, & (28)\\ R_\mu &= (1 - P_{\mu s})(f_{NC,\mu} + f_{\tau,\mu}) + P_{\mu\mu}(1 - f_{\tau,\mu}) + P_{\mu e}(f_{e,\mu} - f_{\tau,\mu})\,, & (29)\\ R_e &= (1 - P_{\mu s})(f_{NC,e} + f_{\tau,e}) + P_{\mu\mu}(f_{\mu,e} - f_{\tau,e}) + P_{\mu e}(1 - f_{\tau,e})\,. & (30) \end{aligned}$$

The general solution for the probabilities is somewhat messy, but if we assume that no other processes contaminate the ν_μ CC signal (i.e., $f_{j,\mu} \simeq 0$) and the ν_e CC events do not contaminate the other signals ($f_{e,j} \simeq 0$), (see Sec. 3), then $P_{\mu\mu} = R_\mu$ and we can invert Eqs. 28 and 30 to obtain

$$P_{\mu s} = 1 - \frac{R_{NC}(1 - f_{\tau,e}) + R_e f_{\tau,NC} + R_\mu\left[f_{\tau,NC}(1 - f_{\mu,e}) - f_{\mu,NC}(1 - f_{\tau,e})\right]}{1 + f_{\tau,NC}(1 + f_{NC,e}) - f_{\tau,e}}\,. \quad (31)$$

The calculation of $\delta P_{\mu s}$ is straightforward.

If no other processes contaminate the ν_e CC signal (i.e., $f_{j,e} \to 0$), then $P_{\mu e} = R_e$ and we obtain

$$\frac{P_{\mu s}}{\delta P_{\mu s}} = \frac{1 + f_{\tau,NC} - R_{NC} + R_\mu (f_{\mu,NC} - f_{\tau,NC}) - R_e f_{\tau,NC}}{\sqrt{(\delta R_{NC})^2 + (f_{\mu,NC} - f_{\tau,NC})^2 (\delta R_\mu)^2 + f_{\tau,NC}^2 (\delta R_e)^2}} . \tag{32}$$

2.3.3 Above τ threshold with a τ measurement

If R_τ is also measured, in addition to R_e, then there are four measurements (R_μ, R_{NC}, R_e and R_τ), but there are only three independent quantities (since $P_{\mu\mu} + P_{\mu e} + P_{\mu\tau} + P_{\mu s} = 1$). One possible approach would be to assume that $P_{\mu s}$ is independent of the other probabilities and use these four measurements to test probability conservation. We do not pursue this option here. Instead, we use probability conservation to eliminate one of the probabilities and use three of the four measurements to determine $P_{\mu s}$ (the fourth measurement could be used to check probability conservation afterwards). Since $P(\nu_\mu \to \nu_\tau)$ is most likely much larger than $P(\nu_\mu \to \nu_e)$ in the L/E regime we are considering, we use R_τ as the third measurement (along with R_μ and R_{NC}). Then the appropriate formulas for the measurables R_{NC}, R_μ and R_τ can be found by the interchange $\tau \leftrightarrow e$ in Eqs. 28–30.

3 Detector simulations

We wish to explore how well in principle a neutrino three-flavor unitarity test can be performed with a given muon-neutrino beam as a function of dataset size, and study which systematic uncertainties are likely to be important, and their impact.

We consider first a "perfect" experiment in which the sensitivity of the unitarity test is determined only by the statistical uncertainties, calculated using a parameterization of the known beam flux and spectrum, together with a simulation of neutrino interactions in the detector. An event simulation is used to determine the relevent detection efficiencies and misidentification factors. We use the NEUGEN Monte Carlo code [12] to simulate neutrino interactions in the detector. Events are classified as ν_μ CC, ν_e CC, or NC. In practice the requirements used to identify events of a given type will depend upon the detector technology. For example, for a water cherenkov detector in our simple analysis we will define a ν_e CC event candidate as an event with an electron candidate above threshold. An electron candidate is either a real electron or a π^0 with an energy exceeding 1 GeV (in which case the two daughter photons from the high energy π^0 produce cherenkov rings that overlap in the detector and cannot be distinguished from a single electromagnetically showering particle). A NC event candidate would be an event containing a π^0 candidate but no muon candidate, where a π^0 candidate has two e-like rings above threshold (which come from a π^0 with energy less than 1 GeV). The definition of CC and NC events can of course be varied, and then tuned to give favorable values for the signal efficiencies and mis-identification factors. Examples are shown in Table 1.

3.1 K2K-like and T2K-like Experiments

To identify the most important systematic uncertainties it is useful to compare the sensitivity of our "perfect experiment" with that of a realistic experiment. We begin with the K2K experiment. K2K uses a beam from the KEK laboratory in Japan. The neutrinos in the KEK beam have a mean energy of 1.3 GeV [9], and the neutrinos travel 250 km to the Super-K water Cerenkov detector. A new experiment T2K is being planned that will exploit a more intense neutrino source that is presently under construction at Tokai, Japan. T2K will also use the Super-K detector, but with a slightly longer baseline (300 km) and narrow-band beam with an axis displaced slightly from pointing directly at the far detector (an "off-axis" beam). The real experimental sensitivities of the K2K and T2K experiments can only be determined by the experimental collaborations. In the following we use the NEUGEN Monte Carlo program to simulate neutrino interactions together with a simple model for the response of a Super-K-like detector. Although this is inadequate to precisely predict the real K2K and T2K sensitivities, it does enable us to identify the dominant sources of systematic uncertainties, and hence explore how the experimental results will depend upon the sizes of these systematics. We use the following parameterization of a Super-K-like detector response:

(a) A threshold of 197 MeV/c for the detection and measurement of muons [14], and 100 MeV/c for electrons and π^0's. These thresholds approximate those used for the atmospheric neutrino analysis of Super-K [15, 14].

(b) Energy resolutions given by [15]

$$\frac{\Delta E_{rms}}{E} = 0.005 + \frac{0.025}{\sqrt{E(\mathrm{GeV})}}\,, \tag{33}$$

for electrons and π^0's and

$$\frac{\Delta p_{rms}}{p} = 0.03\,, \tag{34}$$

for charged pions and muons.

In addition, we use a parametrization of the spectra for the K2K and T2K neutrino beams.

In our analysis we will use only simulated events with visible energy greater than 0.1 GeV. For our "basic" signals we define a ν_μ CC event candidate as an event with a single muon-like ring, a ν_e CC event candidate as an event with a single e-like ring, and a NC event candidate as an event with two e-like rings, which are assumed to be two photons from a single π^0 decay. Given these definitions, the detector efficiencies and mis-identification factors determined from our simulations are listed in Table 1. As shown in the table, the efficiencies ζ_{jj} are of order one-half, and there is no significant contamination of one signal by another due to mis-identification. Also shown are the results of a more aggressive signal definition, where a simulated event with an odd number of e-like rings is labeled as a ν_e CC event candidate, and the remaining events (those with an even number of e-like rings) are labeled as ν_μ CC event candidates if they have one or more μ-like rings or NC if they do not. In this more aggressive scenario no events are discarded, i.e., all events were used for one of the targeted signals. Although some of the misidentification factors are slightly larger for the aggressive scenario, overall they are not greatly changed, while there is a significant improvement in the efficiencies for the CC events.

To investigate whether our analysis is sensitive to the assumed details of the neutrino spectrum we have repeated the calculation of efficiencies and misidentification factors for a K2K-like experiment with a beam that has the same average energy and

Table 1: Signal efficiencies (ζ_{jj}) and normalized mis-identification factors ($f_{i,j}$) in selected long-baseline experiments.

Experiment	j (channel)	Signal	ζ_{jj}	$f_{NC,j}$	$f_{\mu,j}$	$f_{e,j}$	$f_{\tau,j}$
K2K-like	NC	two e-like, no μ-like	0.391	–	0.068	0.052	–
(basic)	μ	one μ-like, no e-like	0.520	0.087	–	0.0007	–
	e	one e-like, no μ-like	0.497	0.003	0.0004	–	–
K2K-like	NC	even e-like, no μ-like	0.437	–	0.078	0.060	–
(aggressive)	μ	even e-like, ≥ 1 μ-like	0.989	0.086	–	0.003	–
	e	odd e-like	0.993	0.002	0.005	–	–
K2K-like	NC	even e-like, no μ-like	0.494	–	0.081	0.011	–
(Gaussian	μ	even e-like, ≥ 1 μ-like	0.994	0.073	–	0.0007	–
beam)	e	odd e-like	0.999	0.0003	0.0004	–	–
T2K-like	NC	even e-like, no μ-like	0.420	–	0.25	0.006	–
(Gaussian	μ	even e-like, ≥ 1 μ-like	0.988	0.036	–	0.0014	–
beam)	e	odd e-like	0.944	0.00002	0.00001	–	–
MINOS-like	NC	no $\mu > 1$ GeV	1.000	–	0.903	–	0.429
$P_{\mu e} = 0$	μ	any $\mu > 1$ GeV	0.749	0	–	–	0
MINOS-like	NC	no e, μ, or γ	0.520	–	1.067	0.005	0.347
$P_{\mu e} \neq 0$	μ	any $\mu > 1$ GeV	0.749	0	–	0	0
	e	no $\mu > 1$ GeV, ≥ 1 e or γ	0.999	0.125	0.090	–	0.064

beam spread as the KEK beam, but with a Gaussian energy spectrum (no long high-energy tail). For the Gaussian beam, the misidentification factors involving ν_e were greatly reduced (since backgrounds from the high-energy tail are now suppressed), but $f_{\mu,NC}$ and $f_{NC,\mu}$ were only slightly affected. Since $f_{\mu,NC}$ is the dominant f factor for a K2K-like experiment, we conclude that our results are not very sensitive to the detailed beam spectrum we assume.

We now consider a T2K-like experiment, where we have used a beam spectrum that corresponds to a detector 2 degrees off-axis. The resulting mis-identification factors for a T2K-like experiment are shown in Table 1. All of the misidentification factors are reduced except for $f_{\mu,NC}$, which is now 0.25. Therefore, in both the K2K-like and T2K-like experiments, the most important contamination is ν_μ CC events being mis-identified as NC events.

3.2 A MINOS-like experiment

The MINOS experiment is a long-baseline oscillation exeriment that will use a neutrino beam from the Fermilab Main Injector and an iron-scintillator sampling calorimeter 730 km away in Minnesota. MINOS is expected to begin data taking early in 2005 with the so-called Low Energy NuMI horn configuration. With a beam energy that is about a factor of three higher than the KEK beam, and a detector that is very different from the water cerenkov detector used by K2K and T2K, the efficiencies and misidentification factors for MINOS will be very different than those for the experiments in Japan. To compute the numbers given in Table 1 we have used a parametrization of the NuMI neutrino beam spectrum for the Low Energy horn configuration, the NEUGEN Monte Carlo Program to simulate neutrino interactions in an iron detector, and a simple parametrization of the response of a MINOS-like detector. In particular we assume:

(a) An energy threshold of 50 MeV for the detection and measurement of electrons, and charged and neutral pions, and a threshold of 1 GeV for the identification and measurement of muons. Note that the MINOS detector is expected to be able to determine the charge and measure the momenta of muons from

0.5 GeV/c to 100 GeV/c, and to distinguish ν_μ CC events from NC events if the muons have momenta exceeding about 1 GeV/c [16]. Our final results are insensitive to the exact values chosen for the energy thresholds; the most dramatic changes occur in $f_{e,NC}$, which doubles in size when the thresholds are increased to 100 MeV. However, $f_{e,NC}$ is very small and consequently does not significantly affect our results. The significant f factors change at most by 5%.

(b) Energy resolutions given by

$$\frac{\Delta E_{rms}}{E} = \frac{0.23}{\sqrt{E(\mathrm{GeV})}}\,, \tag{35}$$

for electrons and π^0's,

$$\frac{\Delta E_{rms}}{E} = \frac{0.55}{\sqrt{E(\mathrm{GeV})}}\,, \tag{36}$$

for charged pions, and

$$\frac{\Delta p_{rms}}{p} = 0.05\,, \tag{37}$$

for muons. Note that in practice the muon energy resolution for the MINOS experiment is expected to be somewhat better (worse) than described by Eq. 37 if the muon ranges out (does not range out) in the detector. We found that $\Delta p_{rms}/p$ values as high as 0.10 do not appreciably change our results.

As shown in the table, for a MINOS-like experiment there is a very large contamination of the NC channel by ν_μ CC events, and mis-identification of ν_τ CC events as NC events is also significant. The efficiency for identifying NC events is about one-half, similar to the K2K-like and T2K-like experiments[1].

The size of $f_{\mu,NC}$ may be understood as follows: f depends not only on the mis-identification fraction, but also on the relative cross sections and efficiency of the signal. For $f_{\mu,NC}$ in the case where electron neutrinos are being ignored, the

[1] Although to first order for MINOS all events with an electron or photon candidate will be classified as NC events, there are three independent probabilities, and it is necessary to extract a separate ν_e signal, in addition to ν_μ and NC signals, to be able to solve for all of the probabilities. Hence we must try to select genuine ν_e interactions from the large NC background. In the table we also show mis-identification factors when all non-μ events are classified as NC events, which could be used when $P_{\mu e}$ is very small.

probability of a μ event being mis-identified as a NC event is about 0.25 and the μ cross section is about 3.5 times larger than the NC cross section, which leads to $f_{\mu,NC} \simeq 0.9$.

We have not considered the effect of τ decays into muons or electrons, each of which occurs with branching fraction of about 18%. If all of the τ decays to muons were identified as muons, then $f_{\tau,\mu}$ would be at most about 0.08; this only has a small effect because $f_{\tau,\mu}$ is proportional to $\sigma_\tau/\sigma\mu \sim 0.3$. In practice many muons from τ decays would not pass the muon minimum energy cut, and $f_{\tau,\mu}$ would be even smaller. Since $f_{\tau,\mu}$ is much smaller than $f_{\mu,NC}$ and $f_{\tau,NC}$, we do not consider the effects of τ decays.

4 Results

4.1 A perfect detector

We first find the sensitivity of the NC unitarity test for a perfect detector, i.e., a detector that can categorize each event correctly as CC muon or NC, with no mis-identification and 100% efficiency. The figure of merit for a perfect detector is given by Eq. 17 with $\zeta_{NCNC} = 1$. We show the 3σ sensitivity for $P_{\mu s}$ (i.e., the minimum value of $P_{\mu s}$ for which $P_{\mu s} = 3\delta P_{\mu s}$) versus N_μ^0 (the number of CC muons expected in the detector with no oscillations) for several values of the NC systematic error in Fig. 1 (the dotted curves). At low statistics the sensitivity is very poor, and for high statistics the sensitivity approaches the asymptotic limit of $3\epsilon_{NC}/(1+3\epsilon_{NC})$, where $\epsilon_{NC} \equiv \delta N_{NC}^0/N_{NC}^0$ is the fractional NC normalization uncertainty.

4.2 More realistic K2K-like and T2K-like experiments

Next we find the NC sensitivity for the K2K-like detector described in Sec. 3.1 for the case $P_{\mu e} \simeq 0$. We generated 400,000 neutrino events using the NEUGEN simulator, from which the normalized mis-identification factors $f_{x,y}$ were calculated. For a given set of probabilities P_{xy}, the values of R_{NC} and R_μ were calculated, and the corresponding measured value of $P_{\mu s}$ was determined from Eq. 11. The uncertainty on $P_{\mu s}$ was calculated using Eq. 14, assuming the uncertainties δR_{NC} and δR_{μ} are uncorrelated and add in quadrature. The 3σ sensitivity for $P_{\mu s}$ is shown in Fig. 1 (solid curves) for various values of ϵ_{NC} for the case $P_{\mu s} = 1 - P_{\mu\mu}$ (all ν_μ oscillating to ν_s; we will consider cases with nonzero $P_{\mu\tau}$ later). For both low and high statistics the K2K-like 3σ sensitivity can be approximated by

$$P_{\mu s}^{min} \simeq \frac{3(1 + f_{\mu,NC})(\delta R_{NC}/R_{NC})}{1 + 3(1 + f_{\mu,NC})(\delta R_{NC}/R_{NC})} \,, \tag{38}$$

which can be derived from Eq. 16, where factors quadratic in the $f_{x,y}$ are ignored. Since $f_{\mu,NC} \simeq 0.08$ for our K2K-like experiment, the NC sensitivity is at most about 1.08 worse than that of the perfect detector for large numbers of events where the statistical uncertainty becomes negligible compared to the systematic uncertainty. At low statistics the efficiency becomes important and the K2K-like performance will be more than 1.08 worse than a perfect detector.

The K2K-like curves in Fig. 1 are plotted for the simple K2K signals in Table 1. The corresponding curves for the more aggressive K2K-like signals are very similar to the simple case; the improved efficiencies are partially compensated for by the slightly higher value of $f_{\mu,NC}$. Thus the result is fairly insensitive to the exact signal criteria used.

We next consider the effects of nonzero $P_{\mu\tau}$. If we assume $P_{\mu\tau} = 1 - P_{\mu s}$ (i.e., $P_{\mu\mu} = P_{\mu e} = 0$), the curves are very close to those of the perfect detector, since the dominant mis-identification term $f_{\mu,NC}$ does not contribute to R_{NC} when $P_{\mu\mu} = 0$. If both $P_{\mu\mu}$ and $P_{\mu\tau}$ are both nonzero (with $P_{\mu e} \simeq 0$), the results will lie somewhere between the curves for K2K-like and the perfect detector.

Finally, we consider nonzero $P_{\mu e}$, in which case R_e must also be measured and $P_{\mu s}$ is determined using Eq. 27. As discussed in Sec. 2.3.1, if $P_{\mu e}$ is of order 0.1 or less (as indicated by oscillation bounds such as from the CHOOZ reactor), and if the misidentification factors are also of order 0.1 or less, then this case reduces to that where the ν_e are ignored. We have verified this numerically for the K2K-like misidentification factors in Table 1.

In summary, the sensitivity of the K2K-like detector to the NC signal is only slightly worse than that of a perfect detector, with the dominant loss of sensitivity coming from the mis-identification of CC muon events as NC. For comparison, in

Fig. 1 we have also shown sensitivity curves for the T2K-like experiment with a Gaussian beam spread. Since $f_{\mu,NC} = 0.25$ in this case, the sensitivity is about a factor of $1.25/1.08 = 1.16$ worse than for K2K.

4.3 The MINOS-like detector

For the MINOS-like case, we generated 320,000 neutrino events using the NEUGEN simulator, and calculated the corresponding mis-identification factors. The 3σ sensitivity for $P_{\mu s}$ was calculated as described above for the case $P_{\mu s} = 1 - P_{\mu\mu}$; the results are shown in Fig. 1. At low statistics, the MINOS-like experiment does better than the K2K-like and T2K-like experiments because of the higher NC efficiency, but at high statistics it does worse because of the larger mis-identification factors.

4.4 Exclusion limit when $P_{\mu s} = 0$

If a 3σ signal for $P_{\mu s}$ is not observed, then an exclusion limit (upper bound) for $P_{\mu s}$ can then be obtained. The 90% C.L. exclusion limit for $P_{\mu s}$ is shown in Fig. 2 for a perfect detector (dotted curves), K2K-like with basic signals (solid curves), and T2K-like with basic signals (dashed curves). To model realistic oscillation probabilities we have assumed a three-neutrino model assuming the parameters $\delta m^2_{31} = 2.0 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1.0$, and $\sin^2 2\theta_{13} = 0.1$. At high statistics the relative values of the exclusion limits are approximately proportional to $(1 + f_{\mu,NC})$, similar to the 3σ sensitivity levels calculated previously. As was the case for the 3σ sensitivity, the MINOS-like detector does better than the K2K-like and T2K-like detectors at low statistics, due to the higher NC efficiency, but not as well at high statistics due to larger misidentification factors.

5 Summary

At low statistics ($\lesssim 1000$ events), experiments with a larger NC efficiency, such as our MINOS-like example, tend to have better sensitivity to the sterile oscillation probability $P_{\mu s}$. At high statistics, the sensitivity in the cases we considered is primarily limited by the systematic uncertainty in the NC rate, ϵ_{NC}, and the contamination of the NC signal from CC μ events, $f_{\mu,NC}$ (and NC contamination from CC τ events, $f_{\tau,NC}$, above τ threshold). The best anticipated ϵ_{NC} is of order a few per cent, so the best 3σ sensitivity and 90% C.L. exclusion limits that can be expected for the sterile oscillation probability will be of order 0.10–0.15 (0.2–0.3 for the oscillation amplitude). The lowest contamination rates are realized for the K2K-like and T2K-like cases. Very fine-grain detectors, such as a liquid Argon TPC, will be subject to much less event mis-identification than the K2K-like or MINOS-like detectors. However, even when event mis-identification is eliminated, at most an 8% improvement is possible over the K2K-like detectors. Therefore, significant improvements in these sterile probability sensitivies or limits can only be achieved by lowering the uncertainty in NC cross sections or improving the event selection criteria, both of which could prove to be challenging but very worthwhile.

Acknowledgments

We thank D. Harris for a critical reading of the manuscript and H. Gallagher for assistance with NEUGEN. VB thanks the Aspen Center for Physics for hospitality during the completion of this paper. This research was supported in part by the U.S. Department of Energy under Grant Nos. DE-FG02-95ER40896, DE-AC02-76CH03000 and DE-FG02-01ER41155, and in part by the Wisconsin Alumni Research Foundation.

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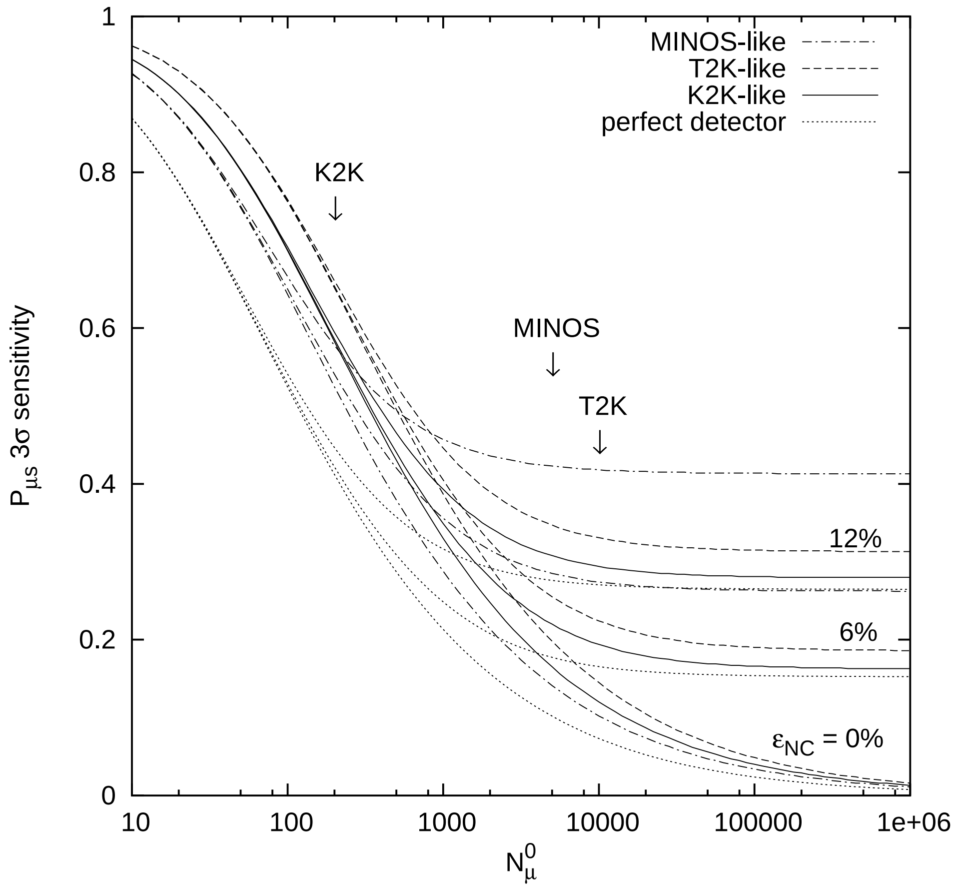

Figure 1: Assuming Gaussian statistical uncertainties, the 3σ sensitivity for measuring $P_{\mu s}$ versus N_μ^0 for fixed values of the NC systematic error for a perfect detector (dotted curves, using Eq. 17), the K2K-like experiment with our basic signal definition (solid), the T2K-like experiment (dashed), and the MINOS-like experiment (dash-dotted). The number of NC events without oscillations is $N_{NC}^0 = 0.156 N_\mu^0$. The systematic uncertainties $\delta N_\mu^0/N_\mu^0$ and $\delta N_e^0/N_e^0$ are assumed to be 2%, except when $\epsilon_{NC} \equiv \delta N_{NC}^0/N_{NC}^0 = 0$, in which case they are 0. The arrows indicate the approximate statistical sensitivities expected for the K2K, T2K and MINOS experiments.

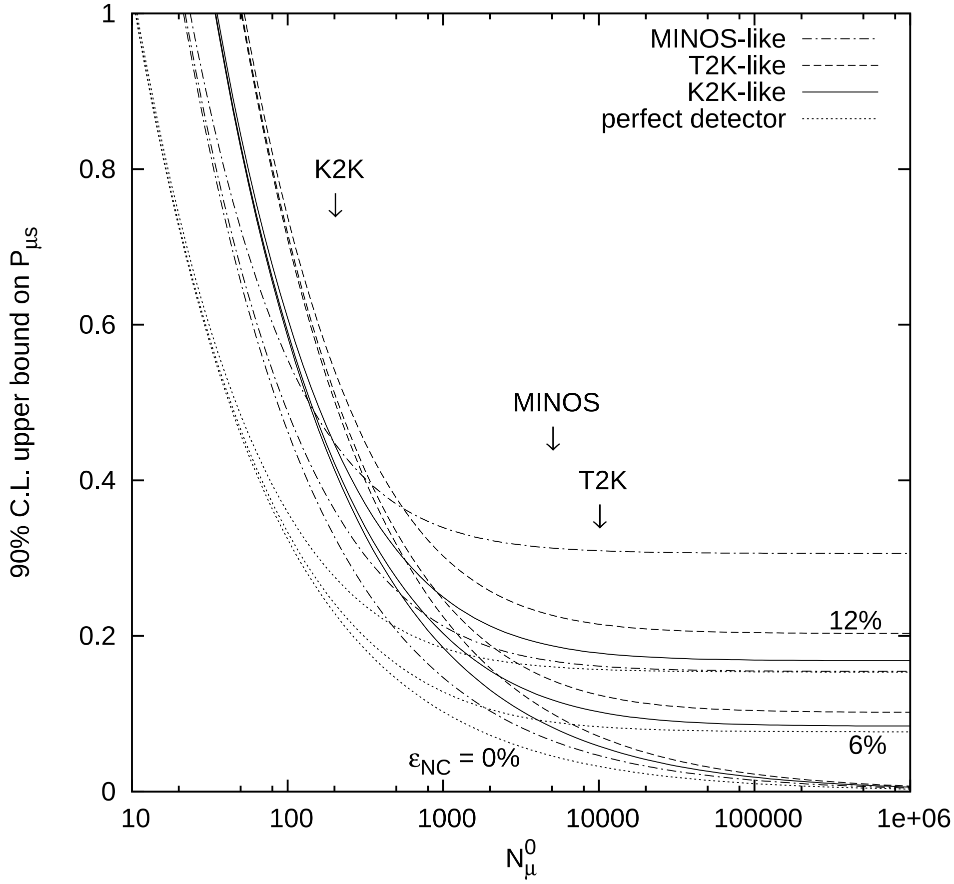

Figure 2: Assuming Gaussian statistical uncertainties, the 90% C.L. exclusion limit for $P_{\mu s}$ versus N_μ^0 for fixed values of the NC systematic error for a perfect detector (dotted curves), the K2K-like experiment with basic signals (solid), the T2K-like experiment with basic signals (dashed), and the MINO-like experiment (dash-dotted). Other assumptions are the same as in Fig. 1.